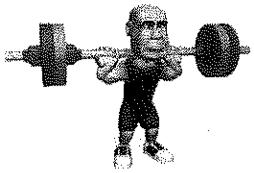
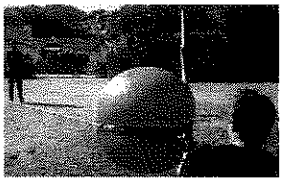
Physics 20 Unit C - UCM, Work and Energy

Work and 3 Kinds of Energy





POS Checklist:

- ☐ recall work as a measure of the mechanical energy transferred and power as the rate of doing work
- ☐ determine, quantitatively, the relationships among the kinetic, gravitational potential and total mechanical energies of a mass at any point between maximum potential energy andmaximum kinetic energy

Energy is a very difficult subject to get started with. You have already learned a lot about this topic in previous science classes, but a lot of what you have learned is probably...not overly helpful.

For example, can anyone tell me *what energy is* ?

Energy is the ability to do work.

I bet y
say.

Saying that energy is "the ability to do work" is only helpful if we know what work is.

So, can anyone tell me what *work* is ?

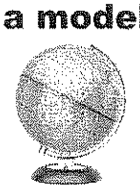
Work is a change in energy.

I bet you say...

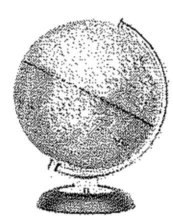
This is called circular reasoning and it's not very helpful...

Today, I'm going to tell you what energy *really* is...

Energy is a model.



Just like a model boat. Or a globe. A globe tells you where oceans and countries are, but a globe is not the *real* Earth, it just helps you understand it better.



The crazy thing about Energy (or any scientific model) is that it's not even really *true.*

It's like asking "is a globe true?". Well, it's true in the sense that Canada on a globe is North of the USA, but it's not *true* in the sense that the Earth is exactly like a globe.

A globe is just a model of the Earth, it's not actually the Earth. But, it's a good model, because it tells you a lot about what the Earth is like. And Energy, as it turns out, is a crazy good model for telling us what Nature is like.

It's tempting to think of energy as some sort of glowing, swirling, weird *stuff*, but really the idea of energy is just another physics model.

Energy is a way of understanding and predicting things that happen around us.

As we go through this topic, we will see how energy can do for us some of the same tasks kinematics and/or dynamics could do, and how it will do new tasks!

In P20: Energy is the ability to cause a:

- change in shape
- change in velocity
- change in location

of an object.

Note that these sound like very Unit A-ie ideas.

Central to the idea of energy is work. Work takes place anytime energy is transferred. Mathematically, we can write this as

$$W = \Delta E$$

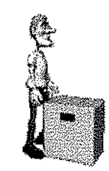
where:
W = work E = energy

This is a huge statement.

$$W = \Delta E$$

So, what does this really mean?

Basically, it means that whenever work takes place, there is a change in energy of the object experiencing the work.



So, if I do work on a box by lifting it up, the work I did is equal to the amount of energy the box gets (its ΔE).

You might recall we had another way of thinking of work back in Science 10:

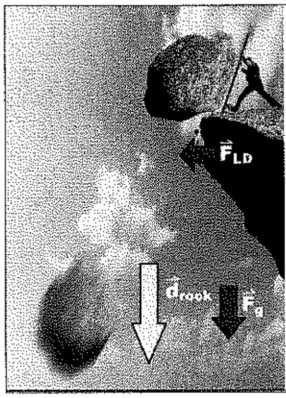
$$W = \vec{F}\vec{d}$$

where:
F = force d = displacement

This gives us the idea that work is what happens when you push on an object with a force and it moves over a displacement.

But, this isn't the whole story...

If I pushed a huge bolder off a cliff, and it gained a bunch of energy as it fell to the ground, would you say that I somehow did the work to give it all that energy?



I might have gotten it started, but I pushed the rock to the left. It moved mostly downwards. Gravity did the work here, not me.

Work is a scalar, meaning it only has magnitude. However, the directions of the force and displacement are important.

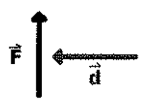
The force must have a component that is colinear to the displacement in order for work to be done.

Colinear - vectors that are in the same dimension, either

parallel vectors

antiparallel vectors

If the force and displacement are perpendicular, then no work is done.



This idea can be modelled mathematically as:

$$W = |\vec{F}||\vec{d}|\cos\theta$$

where θ is the angle between the force and the direction of motion.

$$W = |\vec{F}||\vec{d}|\cos\theta$$

where θ is the angle between the force and the direction of motion.

These vertical lines are special mathematical symbols meaning *absolute value.* The lines tell you that you need to take the negative sign off any number you sub in for force or displacement.

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ex) Determine the work done if

a) force = 30 N [W], displacement = 7.0 m [E]

$\vec{F} = 30N$ $\vec{d} = 7m$ $180°$

$W = |\vec{F}||\vec{d}|\cos(\theta)$

$W = (30N)(7m)\cos(180°)$

$W = 210J \rightarrow 2.1\times10^2 J$

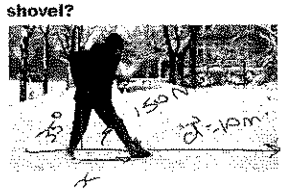
b) force = 30 N [W], displacement = 7.0 m [S]

$\vec{F} = 30N$ $90°$ $7.0m = \vec{d}$

$W = (30N)(7m)\cos(90°)$

$W = 0 J$

Ex) Katie is shoveling the walk. A force of 150 N is applied down the shovel handle, which makes an angle of 35.0° with the horizontal. Katie pushes the shovel 10.0 m. How much work is being done on the shovel?



$W = |\vec{F}||\vec{d}|\cos\theta$

$= (150N)(10m)\cos(35°)$

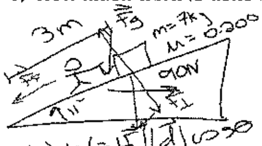
$= 1229 J$

$= 1.23\times10^3 J$

a) How much work does Adele do?

b) How much work is done by gravity against the crate?

c) How much work is done by friction against the crate?



a.) $W = |\vec{F}||\vec{d}|\cos\theta$

$W = (90N)(3m)\cos(11°)$

$W = 265J$ (Adele)

$= \underline{2.7\times10^2 J}$

b.) $W = |\vec{F}||\vec{d}|\cos\theta$

$W = (7kg)(9.81)(3m)\cos(79°)$

$W = \underline{39.3J}$ (gravity)

c.) $\vec{F}_f = \mu \vec{F}_N$

$= (0.200)(7kg)(9.81m/s^2) \times \cos(11°)$

$\vec{F}_\perp$

$\vec{F}_f = 13.4817N$

$W = (13.4817N)(3m)\cos(180°)$

$W = \underline{40.4J}$ (friction)

HW: Read pg 293-294
Practice Problems on page 294

3 Kinds of Energy

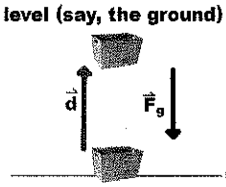
There are many different forms of energy, but none of them are distinct. We simply have different ways of determining how much energy is present in different situations.

Types of energy:

- **P20 - Mechanical Energy (kinetic and gravitational and elastic potential)**
- **P30/S30 - Nuclear**
- **C30/S30 - Chemical Potential**
- **P30 - Electric Potential**
- **S10/C30 - Thermal**
- **P30 - Electromagnetic**

Gravitational Potential Energy

Imagine raising an object to some height above a reference level (say, the ground) which we will call zero.



The force of gravity will be colinear to the displacement, which means it must take work to elevate an object.

If there is work done on an object, we say the object has gained some amount of energy. Work means there was a change in energy. The energy gained by raising an object against the gravitational field of the earth is called <u>potential energy (E_p or PE)</u>.

Lets derive an equation for E_p:

$$W = \Delta E$$

$$Fd = \Delta E$$

$$mgd = \Delta E$$

As we call the distance moved the height, h, in metres.

$$\boxed{mgh = E_p}$$

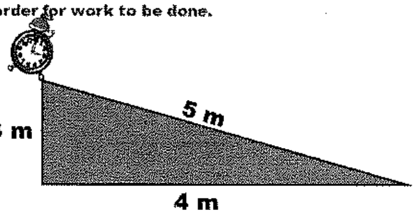
Where E_p = potential energy, measured in joules (J).

Question: where did all the vector arrows go on the variables here?

$$E_p = mgh$$

Remember:

The height we consider here is the vertical height above the ground the object is raised through. This displacement must be colinear to the force of gravity in order for work to be done.



Reference Levels

The height you measure is always with respect to some reference level. The reference may be the floor, the table, or some other point in space.

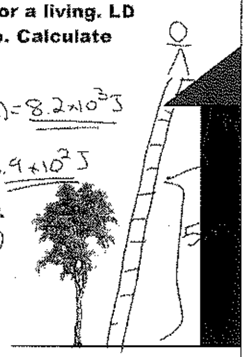
Where we measure from is not terribly important as we are just calculating the change in energy from one point to another.

ex) Young LD (m = 70 kg) climbed ladders for a living. LD climbed a 12 m ladder on one particular job. Calculate LD's E_p with respect to:

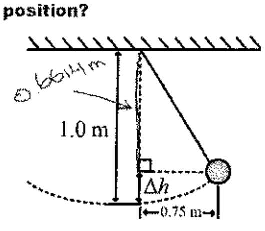
a) The ground.
$E_p = mgh$
$= (70kg)(9.81m/s^2)(12m) = \underline{8.2 \times 10^3 J}$

b) The roof (11 m above the ground)
$E_p = (70kg)(9.81m/s^2)(1m) = \underline{6.9 \times 10^2 J}$

c) A tree, 7.0 m below the top of the ladder.
$E_p = (70kg)(9.81m/s^2)(7.0m)$
$= \underline{4.8 \times 10^3 J}$



ex) A pendulum bob of mass 2.0 kg is fixed from the ceiling by a string of length 1.0 m. If the bob is pulled 0.75 m to one side, what is its gravitational potential energy with respect to its equilibrium position?



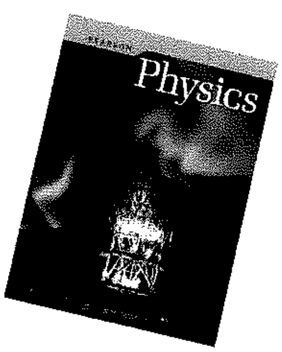
$E_p = mgh$
$= (2kg)(9.81m/s^2)(0.3386m)$
$\boxed{E_p = 6.6J}$

$c^2 = a^2 + b^2$
$(1.0m)^2 = (0.75m)^2 + b^2$
$b = 0.6614m$

1.0m, b, 0.75m

$\Delta h = 1.0m - 0.6614m$
$\Delta h = 0.3386m$

Practice Problems:
Page 298 #1 - 3
(more notes to come)



Kinetic Energy

Energy of Motion (kinetic is Greek for "motion")

Kinetic energy is the energy needed to accelerate a body to a certain speed. In P20, we only deal with constant acceleration, and we assume that the object starts at rest (v_i = 0 m/s).

$E_k = \frac{1}{2} mv^2$

Where:

v = final speed (recalling that the object starts at rest) in m/s
m = mass (kg)

Sometimes an object does not start at rest. In these cases, the object already has some kinetic energy, E_{ki}, and then gains some more energy (by having work done upon it) to go to a final kinetic energy, E_{kf}.

The work done on the object causes a change in energy.

This statement is the basic premises of the Work-Energy theorem, which we study in more detail later.

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ex) An 8.0 kg rock is dropped from a height of 7.0 m. What is the kinetic energy of this rock as it hits the ground?

$\vec{v}_f^2 = \cancel{\vec{v}_i^2} + 2\vec{a}\vec{d}$

$\vec{v}_f^2 = 2(-9.81 m/s^2)(-7.0 m)$

$\vec{v}_f = -11.7192 m/s$

$E_k = \frac{1}{2} mv^2$

$E_k = \frac{1}{2}(8 kg)(+11.7192 m/s)^2$

$E_k = 5.5\times10^2 J$

$E_p = mgh$

$E_p = (8 kg)(9.81 m/s^2)(7 m)$

$= 5.5\times10^2 J$

---

ex) A 10.0 N melon is accelerated uniformly from rest at a rate of 2.50 m/s². What is the kinetic energy of this object after it has accelerated a distance of 15.0 m?

$E_k = \frac{1}{2} mv^2$

$\vec{F}_g = m\vec{g}$

$10 N = m(9.81 m/s^2)$

$m = 1.019 kg$

$E_k = \frac{1}{2}(1.019 kg)(8.66 m/s)^2$

$\vec{v}_f^2 = \cancel{\vec{v}_i^2} + 2\vec{a}\vec{d}$

$\vec{v}_f^2 = 2(2.5 m/s^2)(15 m)$

$\vec{v}_f = 8.66 m/s$

$E_k = 38.2 J$

$m = 1.019 kg$

$E_k = \frac{1}{2} mv^2$

$E_k = \frac{1}{2} m\left(\sqrt{2\vec{a}\vec{d}}\right)^2$

$= \frac{1}{2} m\, 2\vec{a}\vec{d}$

$E_k = m\vec{a}\vec{d}$

$E_k = \vec{F}\vec{d}$

---

ex) By what factor must the KE of an object be increased to cause the speed to triple?

$E_k = \frac{1}{2} mv^2$

$(E_k) \propto (1)(1)(3)^2$

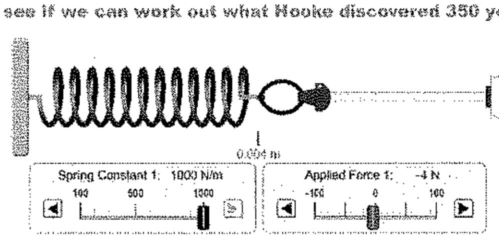
$E_k \propto 9$

The E_k would need to be 9 times greater.

Hooke's Law and Elastic Energy

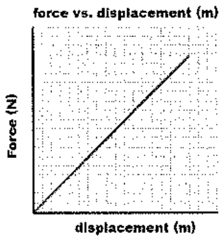
In 1676, Robert Hooke devised a relationship between the amount of stretch in a spring and the weight suspended by that spring.

Let's see if we can work out what Hooke discovered 350 years ago!



https://phet.colorado.edu/sims/html/hookes-law/latest/hookes-law_en.html

When a graph of force vs. displacement was made, a linear relationship is established.



This graph is a linear function of y-int = 0.

From this linear equation, we get:

Hooke's Law

$$\vec{F} = -k\vec{x}$$

where: F = force applied to the spring (N)
x = displacement from equilibrium (stretch or compress) of spring (m)
k = spring constant (slope) (N/m)

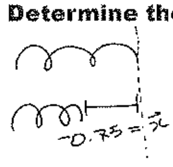
Note:

Hooke's Law appears with a negative sign in front of the spring constant on your formula sheet:

$$\vec{F}_s = -k\vec{x}$$

because the force applied by a spring is always in the opposite direction of the displacement.

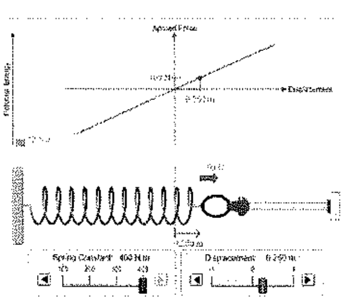
ex) A spring with a spring constant of 3.5 N/m is compressed 75 cm to the left.
a) Determine the force in the spring.



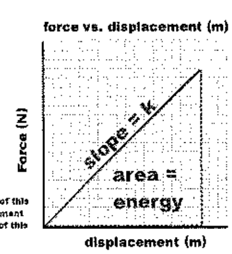
$\vec{F}_s = -k\vec{x}$
$= -(3.5\frac{N}{m})(-0.75m)$
$\vec{F}_s = +2.6N$

Hooke also worked out how to calculate the energy stored in a spring. Let's see what that relationship looks like:

https://phet.colorado.edu/sims/html/hookes-law/latest/hookes-law_en.html



To get the energy from the graph of force vs. displacement, Hooke found the area under the graph.



$$A = 1/2\ bh$$

$$E = 1/2\ Fd$$ since the "base" of this graph is displacement and the "height" of this graph is force

$$E = 1/2\ {-kxx}$$ since since force in a spring is -kx, and the displacement is also called x

$E_p = \frac{1}{2}kx^2$

elastic potential energy

$$E_p = 1/2kx^2$$

Where:

E_p = Potential Energy (J)
k = Spring Constant (N/m)
x = Displacement of Spring (m)

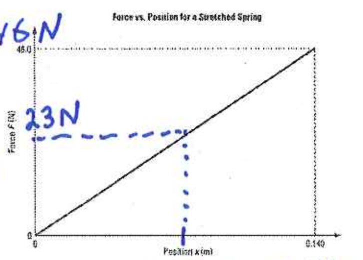
ex) A spring with a spring constant of 3.5 N/m is compressed 75 cm to the left.
b) Determine the elastic potential energy stored in the spring.

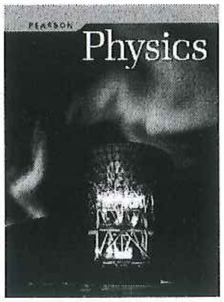
$$E_p = \frac{1}{2}kx^2$$
$$= \frac{1}{2}(3.5\frac{N}{m})(0.75m)^2$$
$$= 0.98J$$

ex) From the graph below:
(a) Calculate the energy stored in the spring when the force is 46.0 N.
(b) Compare the energy in the spring at 0.14 m and 0.070 m.

a.) E_p = Area.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(0.14m)(46N)$$
$$= 3.22J$$





HW: Page 305 #2 - 4, 6 - 9, 11

b.) E_p = Area.

$$A = \frac{1}{2}(0.070m)(23N)$$
$$= 0.805J$$ ← 4 times less!

← makes sense, as this even looks like 4 times less area on the graph!