



## Mechanical Energy and the Work Energy Theorem

### POS Checklist:

- define mechanical energy as the sum of kinetic and potential energy.
- determine, quantitatively, the relationships among the kinetic, gravitational potential and total mechanical energies of a mass at any point between maximum potential energy and maximum kinetic energy
- analyze, quantitatively, kinematics and dynamics problems that relate to the conservation of mechanical energy in an isolated system
- describe, qualitatively, the change in mechanical energy in a system that is not isolated.

Review: pg 305

2. Describe how a non-zero force can act on an object over a displacement and yet do no work.
3. Explain why the frame of reference affects the calculated value of an object's gravitational potential energy but not the change in its gravitational potential energy.

The force and displacement can be perpendicular.

Depending on where you make your "ground", i.e. 0.0m, you will have different values of  $E_p$ .

8. A spring has an elastic constant of 650 N/m. Initially, the spring is compressed to a length of 0.100 m from its equilibrium position.

- (a) What is the elastic potential energy stored in the spring?
- (b) How much further must the spring be compressed if its potential energy is to be tripled?

$$a.) E_p = \frac{1}{2} kx^2$$

$$= \frac{1}{2} (650 \frac{N}{m}) (0.100m)^2$$

$$= \underline{\underline{3.25J}}$$

$$b.) E_p = \frac{1}{2} kx^2$$

$$E_p \propto \frac{1}{2} kx^2$$

$$3 \times (1)(1)x^2$$

$$\sqrt{3} \propto x$$

$$\text{new } x = \sqrt{3} \times 0.100m = 0.173m$$

$$\Delta x = 0.073m \text{ further.}$$

## Mechanical Energy

We have dealt with 3 kinds of energy so far:

$$E_p = mgh$$

gravitational  
potential energy

$$E_k = \frac{1}{2}mv^2$$

kinetic energy

$$E_p = \frac{1}{2}kx^2$$

elastic potential  
energy

These energies are rarely found on their own. In a given situation, there is often more than one type of energy present. Therefore, it is helpful to consider the total of the energies present: this is the mechanical energy.

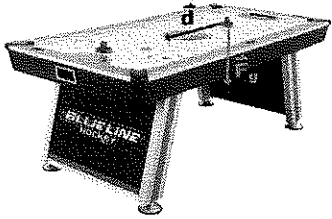
**Mechanical Energy: the sum of kinetic and potential energy in a given system.**

Wait...what's a system?

In Physics, a system is a group of objects that you study. A system could be a table top with a block sliding across it, a brick on a rope, a planet and a moon, etc.

We have two types of systems in Physics:

**Isolated System** - a system in which no outside forces do work (i.e. no outside forces change the energy in the system)



An air hockey table is an example of an isolated system. There is very little force of friction on the pucks as they move, so the only outside force acting on the pucks is gravity. But, since gravity acts downwards, and the puck's displacement is horizontal, no work is done (gravity doesn't add or take away energy from the pucks).

**Non-isolated System** - a system in which outside forces do work (i.e. an outside force changes adds or takes away energy from the system)

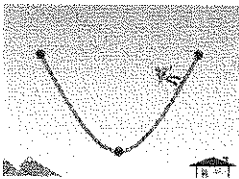
Here, the car and dog are a non-isolated system. An outside force (the girl) is exerting a force which does work (speeds the car up).



In an isolated system, since no forces add or take away any energy from the system, the total energy never changes. This idea is called the

### Law of Conservation of Energy.

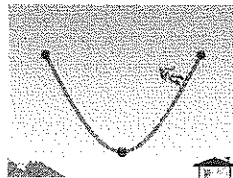
In an isolated system, mechanical energy is conserved. Energy is not created or destroyed, only changed in form.



[https://phet.colorado.edu/sims/html/energy-skate-park-basics\\_en.html](https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

The Conservation of Energy seems to fall apart when we consider friction, which makes the system "lose" energy. But, this energy is still accounted for (although, we can't really convert it back from the thermal energy produced by friction or air resistance).

Forces, like friction or air resistance, are called non-conservative forces. These forces dissipate energy from the system (it can not be recaptured).



This is what happened in the skate park with friction turned all the way up.

Calculate the:

$E_p = mgh$   
 $= (200kg)(9.81m/s^2)(15m)$   
 $= 3.04 \times 10^4 J$

a)  $E_p$  at the first hill.  
 b) The  $E_k$  and speed at the bottom dip.  
 c) The speed at the top of the second hill.

$\frac{1}{2}mv^2$  known

b)  $E_p = E_k$   
 $mgh = \frac{1}{2}mv^2$   
 $2 \cdot (9.81m/s^2)(15.5m) = \frac{1}{2}v^2$   
 $v = 17.4m/s$

c)  $E_p = E_p + E_k$   
 $mgh = \frac{mgh}{m} + \frac{1}{2}mv^2$   
 $(9.81m/s^2)(15.5m) = (9.81m/s^2)(7.35m) + \frac{1}{2}v^2$   
 $12.6m/s = v$

ex) Determine the final velocity of a ball dropped from 15 m.

$v_i = 0m/s$   
 $a = -9.81m/s^2$   
 $\Delta d = 15m$   
 $v_f^2 = v_i^2 + 2a\Delta d$   
 $v_f^2 = 2(-9.81m/s^2)(-15m)$   
 $v_f = \underline{\underline{-17m/s}}$

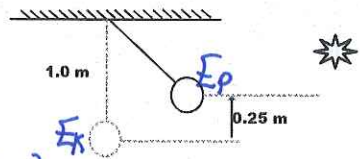
$E_p = E_k$   
 $mgh = \frac{1}{2}mv^2$   
 $(9.81m/s^2)(15m) = \frac{1}{2}v^2$   
 $v = \underline{\underline{17m/s}}$

ex) A pendulum is dropped from the position as shown 0.25 m above equilibrium. What is the speed of the bob as it passes through the equilibrium position?

$E_p = E_k$   
 $mgh = \frac{1}{2}mv^2$

$(9.81 \text{ m/s}^2)(0.25 \text{ m}) = \frac{1}{2}v^2$

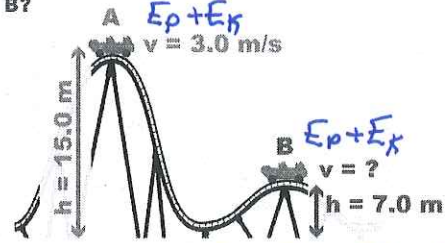
$v = 2.2 \text{ m/s}$



Interesting Observation about the last two questions:

- ① mass cancels out!
- ② You could solve with  $v_f^2 = v_i^2 + 2ad$ .

ex) A roller coaster is traveling on a frictionless track as shown. If the speed of the coaster at A is 3.0 m/s, what is the speed at B?



$E_p + E_k = E_p + E_k$   
 $mgh + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv^2$

$(9.81 \text{ m/s}^2)(15.0 \text{ m}) + \frac{1}{2}(3 \text{ m/s})^2 = (9.81 \text{ m/s}^2)(7.0 \text{ m}) + \frac{1}{2}v^2$

$v = 13 \text{ m/s}$

Mass cancels out!

omit

But, what if the system is non-isolated and an outside force adds some energy to the system? Then, we can rethink the Law of Conservation of Energy to get...

The Work-Energy Theorem

$W = \Delta E_k + \Delta E_p$

All work done on a system is the sum of the changes in potential and kinetic energies.

omit

ex) Farmer LD is hauling feed to his chickens. He lifts a 9.00 kg bucket of feed up 5.00 m by rope to his coop roof. This takes a force of 150 N upwards.

a) What work does LD do on the feed?

$W =$

b) What is the change in  $E_p$  of the feed?

c) What is the change in  $E_k$  of the feed?

omit

ex) At a Toboggan Party, a 150 kg sled and riders are pushed up a hill. The initial velocity of the riders is 2.50 m/s and the final velocity is 5.80 m/s. The hill has a vertical height of 6.53 m. What amount of work is needed to push the sled up the hill?

$W = \Delta E_k + \Delta E_p$

HW: Page 310 # 6, 7, 8

HW: Page 315-316 #1, 3, 4

Read: Page 319-322