Key!

Physics 20 Unit C - Uniform Circular Motion and Energy

Vertical Circular Motion

POS Checklist:

☐ explain, qualitatively, uniform circular motion in terms of Newton's laws of motion

This takes one year

365 days × 24h × 60min × 60s

= 31536000s

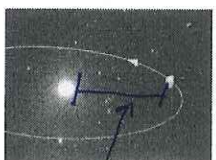
Review:

Assuming the earth makes a circle around the sun at a mean distance of 1.50 x 10⁸ km, determine the speed of the earth's orbit, in km/h.

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi(1.50\times10^{11}\,m)}{31536000\,s}$$

$$v = 2.99\times10^{4}\,m/s$$



$r = 1.50\times10^{11}\,m$

Review:

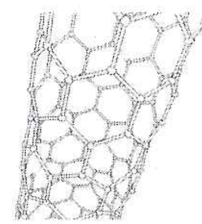
Carbon nanotubes are incredibly strong and lightweight. A thin 1.50 m braided strand of tubes has a theoretical breaking point of 62000 N. Determine the maximum speed a 10 kg mass can be spun at the end of such a rope.

$$\vec{F}_T = \vec{F}_c$$

$$62000\,N = \frac{mv^2}{r}$$

$$62000\,N = \frac{(10\,kg)\,v^2}{1.5\,m}$$

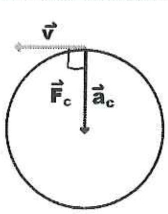
$$v = 96\,m/s$$



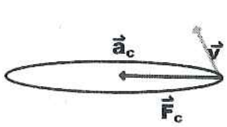
A carbon nanotube, an isomer of carbon.

Vertical Circular Motion

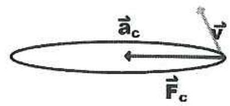
Recall horizontal circular motion:



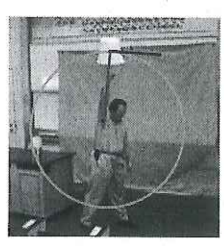
Although we drew the diagram like this, a more accurate way would have been like this:

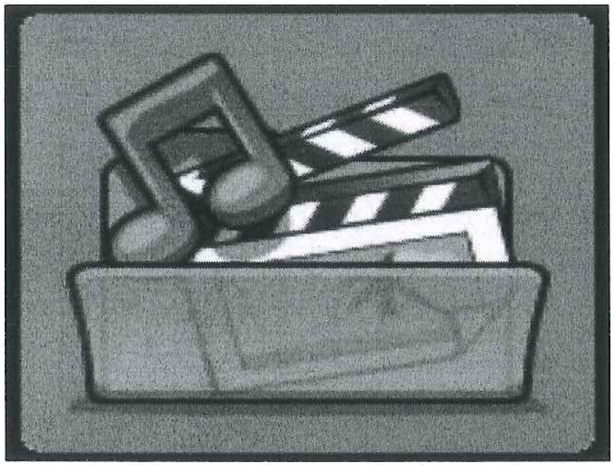


Velocity is at a tangent to the circle, acceleration and force are directed inwards.



In any situation where an object undergoes UCM, the centripetal force is supplied by some other force, like friction, tension, magnetism, electricity...or gravity!

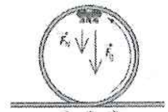




Objects undergoing UCM in vertical paths have gravity (and occasionally another force) supplying the centripetal force. For example: (pg 261)

A roller-coaster in a loop-the-loop:

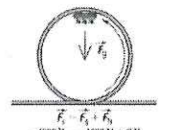
▶ Figure 5.28(a) The first time through the loop, the speed is such that the roller coaster requires a centripetal force of 1500 N to keep it moving in a circular path. At the top of the loop, the roller coaster car will experience a centripetal force that is the sum of the force of gravity and the force exerted by the track, pushing the car inward to the centre of the circle. The centripetal force acts down, so it is −1500 N. The force of gravity is constant at 1000 N so the track pushes inward with 500 N to produce the required centripetal force. The car goes around the loop with no problem.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-1500\ N = -1000\ N + -500\ N$

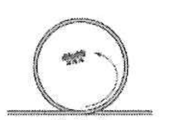
▶ Figure 5.28(b) Suppose the next time the car goes around the track, it is moving more slowly, so that the centripetal force required is only 1000 N. In this case, the force of gravity alone can provide the required centripetal force. Therefore, the track does not need to exert any force on the car to keep it moving on the track. There is no normal force, so the force of gravity alone is the centripetal force. The car goes around the loop again with no problem.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-1000\ N = -1000\ N + 0\ N$

▶ Figure 5.28(c) Now suppose the last time the car goes around the track, it is moving very slowly. The required centripetal force is just 800 N, but the force of gravity is constant, so it is still 1000 N; that is, 200 N more than the centripetal force required to keep the car moving in a circular path with this radius. If the track could somehow pull upward by 200 N to balance the force of gravity, the car would stay on the track. This is something it can't do in our hypothetical case. Since the gravitational force cannot be balanced by the track's force, it pulls the car downward off the track.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-800\ N = -1000\ N + 200\ N$

Since the normal force cannot pull upward, it cannot generate +200 N. 200 N more is needed to keep the car on a track of this radius, so the car falls off.

The centripetal force is supplied by the normal force and gravity at the top of the loop. Knowing this, we can do some calculations:

ex) (pg 262 #1) Neglecting friction, what is the minimum speed a toy car must have to go around a vertical loop of radius 15.0 cm without falling off?

$\vec{F_c} = \vec{F_g}$

$\frac{m v^2}{r} = m\vec{g}$

$\frac{v^2}{0.15m} = 9.81 m/s^2$

$v = \underline{1.21 m/s}$

ex) (pg 262 #2) What is the maximum radius a roller coaster loop can be if a car with a speed of 20.0 m/s is to go around safely?

$\vec{F_c} = \vec{F_g}$

$\frac{m v^2}{r} = mg$

$\frac{(20 m/s)^2}{r} = 9.81 m/s^2$

$r = \underline{40.8 m}$

This is normal force!

ex) What is the force the roller coaster track is providing at the bottom of the loop to a 102 kg cart traveling at 15.0 m/s around a 7.0 m radius loop?

$\vec{F_c} = \vec{F_N} + \vec{F_g}$

$\frac{m v^2}{r} = \vec{F_N} + m\vec{g}$

Negative because $\vec{F_N}$ is up and $\vec{F_g}$ is down!

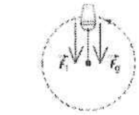
$\frac{(102kg)(15m/s)^2}{(7m)} = \vec{F_N} + (102kg)(-9.81 m/s^2)$

$\vec{F_N} = \underline{4.3 \times 10^3 N}$

This situation can also apply to a bucket-water system: (pg 263)

▶ Figure 5.30(a) The bucket is at the top of the circle. In this position, two forces are acting on the bucket: the force of gravity and the tension of the rope. Both are producing the centripetal force and are acting downward. The equation to represent this situation is: $\vec{F}_c = \vec{F}_g + \vec{F}_T$

(a)



$\vec{F}_c = \vec{F}_g + \vec{F}_T$

▶ Figure 5.30(b) When the bucket has moved to the position where the rope is parallel to the ground, the force of gravity is perpendicular to the tension. It does not contribute to the centripetal force. The tension alone is the centripetal force. We can write this mathematically as:
$\vec{F}_c = \vec{F}_T$

(b)

$\vec{F}_c = \vec{F}_T$

▶ Figure 5.30(c) As the bucket moves through the bottom of the circle, it must have a centripetal force that overcomes gravity. The tension is the greatest here because gravity is acting opposite to the centripetal force. The equation is the same as in (a) above, but tension is acting upward, so when the values are placed into the equation this time, $\vec{F}_T$ is positive and $\vec{F}_g$ is negative. The effect is demonstrated in Example 5.7.

(c)

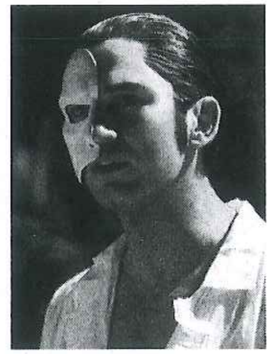
$\vec{F}_c = \vec{F}_g + \vec{F}_T$

Here, the force of gravity and tension work to supply the centripetal force.

Q: So what keeps the water in the bucket?

It would seem to you like the force acting on the water must be acting outward to keep the water in the bucket, but it is not. The centripetal force is pointed towards the centre of the circle. And this force keeps things in a circular pattern, not a pattern where the water will fall.

Some laypersons call the "force" that keeps the water in the bucket the "centralfugal force", but in reality, there is no such thing. This is what us physicists call a phantom force.



ex) A shoelace can hold a force of 135 N before breaking. If a 2.00 kg brick **is tied to the end of this lace (L = 1.10 m), how fast can I spin it vertically before the string breaks?**

The rope will break at the bottom of the loop where the $\vec{F_T}$ and $\vec{F_g}$ are in a "tug of war"!

$\vec{F_c} = \vec{F_T} + \vec{F_g}$

$\frac{mv^2}{r} = \vec{F_T} + m\vec{g}$

as $\vec{F_g}$ is downwards

$\frac{(2kg)v^2}{1.10m} = 135N + (2kg)(-9.81m/s^2)$

$v = \underline{7.97m/s}$

ex) Batman and Robin (circa 1965) are spinning their patented Batarangs in a vertical circle of r = 0.75 m. What is the minimum speed needed to keep the Batarang in UCM?

$\vec{F_c} = \vec{F_g}$

$\frac{mv^2}{r} = m\vec{g}$

$\frac{v^2}{0.75m} = 9.81m/s^2$

$v = 2.7m/s$

HW: pg 268 #6-13 all