KEY!

P20 Unit C: Uniform Circular Motion and Gravitation

Intro to Uniform Circular Motion

POS Checklist:

- ☐ describe uniform circular motion as a special case of two-dimensional motion.
- ☐ explain, qualitatively and quantitatively, that the acceleration in uniform circular motion is directed toward the centre of a circle.
- ☐ explain, quantitatively, the relationships among speed, frequency, period and radius for circular motion.
- ☐ explain, qualitatively, uniform circular motion in terms of Newton s laws of motion

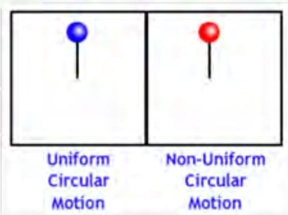
This unit deals with three ideas we've already spent some time considering:

- kinematics
- dynamics
- gravity

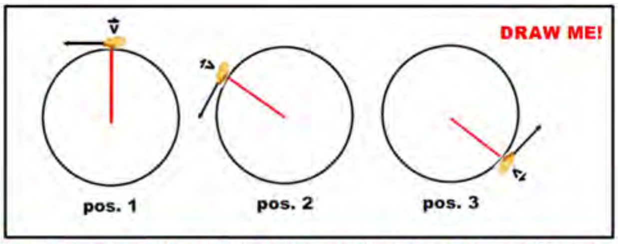
However, up until now, we've only considered objects moving in straight lines. The physics of circles still remains a mystery to you.

Uniform Circular Motion

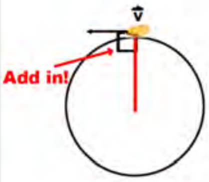
The "uniform" of uniform circular motion comes down to speed.
In UCM, the speed of the objects stays the same, while the velocity constantly changes.



To see how this works, let's draw a picture of a hot pocket on a string, moving in a horizontal circle (this is a top-down view of the motion).



If we examine different the hot pocket's path at three different moments in time, we notice that the direction of the velocity is different. So, even though the speed remains uniform, the velocity is always changing.



The velocity of the object in UCM always acts at a tangent to the circle in the direction of movement. The velocity at any given point in time (called the instantaneous velocity) is also perpendicular to the radius of the circle.

<u>Direction of velocity is perpendicular to the radius, at a tangent to the circle!!</u>

Magnitude of Velocity for UCM

So, speed stays the same, but velocity is always changing. Hmm... a changing velocity? What does that mean?

Recall that in Physics, "a change in" can be represented by the Greek symbol delta (Δ). So we could say that in UCM, we have a $\Delta\vec{v}$. What formulas do we have that involve $\Delta\vec{v}$?

Good Old

$$\Delta\vec{v} = \Delta\vec{d} / \Delta t$$

Now, the value Δt stands for the change in time, in this case, the amount of time it takes for the object to make one complete revolution around in a circle.

In UCM, this time value is called the period, which is represented by a capital letter T.

<u>A period is the amount of time it takes an object to complete one cycle of movement.</u>

So, if we wanted to determine the magnitude of the velocity of our hot pocket, we could substitute in T for Δt.

$$\Delta\vec{v} = \Delta\vec{d} / T$$

But what is Δd? This variable represents the distance the object goes through in its circular path. This displacement is the same as the circumference of the circle which the object traces out.

$$\Delta\vec{d} = 2\pi r$$

We can now write an equation for the velocity of an object moving in a circle of radius r, in a period T:

$$|\vec{v}_c| = \frac{2\pi r}{T}$$

where:

$\vec{v}$ = centripedal velocity (m/s)
r = radius of circle (m)
T = period (s)

The absolute value signs around the centripetal velocity tells us that this is a <u>scalar formula</u>. This formula will only tell us the magnitude of the velocity; we need to get the direction from the context of the question.

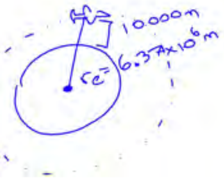
Example: If the Hot Pocket has a period of 1.5 s and the string is 1.25 m long, what is the magnitude of the HP's velocity?

$$|\vec{v}_c| = \frac{2\pi r}{T}$$

$$|\vec{v}_c| = \frac{2\pi(1.25m)}{1.5s}$$

$$|\vec{v}_c| = 5.2 m/s$$

Ex: A jet flys at a height of 10 000 m above the surface of the earth in a circular path around the planet. If the velocity of the jet is 150 m/s, how long does it take the jet to go around the world?



$$|\vec{v}_c| = \frac{2\pi r}{T}$$

$$150\,m/s = \frac{2\pi(6.37\times10^6 m + 10000 m)}{T}$$

$$T = 2.67\times10^5 s$$

$$T = 74.2 \text{ hours}$$

Frequency

Frequency is equal to the number of revolutions or cycles per second. **It is related to the period by the expression:**

$$f = \frac{1}{T}$$

where:
f = frequency (Hz, or s^{-1})
T = period (s)

Ex) A tire balancing machine spins a tire horizontally at a rate of 830 RPM (rotations per minute). If the tire has a diameter of 0.58 m, what is the speed that a point on the outer edge of the tire moving, in m/s?

$$f = \frac{830 \text{ rotations}}{60 s} = \frac{13.8\bar{3} \text{ rotations}}{1 s} = 13.83 Hz$$

$$T = \frac{1}{f} \qquad T = \frac{1}{13.83 Hz}$$

$$T = 0.07229 s$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29 m)}{0.07229 s} = \underline{\underline{25 m/s}}$$

Centripetal Acceleration

The acceleration of an object in UCM can be found using these two equations:

$$|\vec{a}_c| = \frac{v^2}{r} \qquad |\vec{a}_c| = \frac{4\pi^2 r}{T^2}$$

We can sub this formula in to get

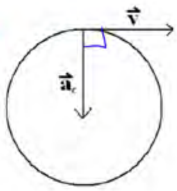
$$|\vec{v}_c| = \frac{2\pi r}{T}$$

Hey, where did the word "centripetal" come from? It was actually coined by our pal Newton, when he worked out the physics of circular motion hundreds of years ago.

Centripetal means "centre seeking".

He called it centripetal acceleration because in UCM, the acceleration always points to the centre of the circle.

*Draw Me!



So velocity acts at a tangent to the circle, perpendicular to the radius, and the acceleration always acts towards the centre of the circle!

ex) The bobsled track in Calgary has turns of radii 33 m and 24 m. During the 1988 Olympic Winter Games, the Jamaican team went through the turns at a speed of 34 m/s.

a) What acceleration did the team experience in the first turn, in m/s^2?

$\vec{a} = \frac{v^2}{r} = \frac{(34\,m/s)^2}{33\,m} = \underline{\underline{35\,m/s^2}}$ towards center.

b) What acceleration did the team experience in the second turn, in g-forces?

$\vec{a} = \frac{v^2}{r} = \frac{(34\,m/s)^2}{24\,m} = \frac{48\,m/s^2}{9.81\,m/s^2} = 4.9$ g's towards center of circle.

To get units of "g-force", express acceleration in multiples of 9.81 m/s^2.

ex) Ski-doo A is traveling in a straight line with v = 18 m/s and is picking up speed at an acceleration of 3.5 m/s^2. Ski-doo B is traveling in a circle with the same velocity and an a_c = 3.5 m/s^2. What will the magnitude of the velocities of each Ski-doo be after 2.00 s?

$A \rightarrow \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$ $\quad 3.5\,m/s^2 = \frac{\vec{v}_f - 18\,m/s}{2.00\,s}$

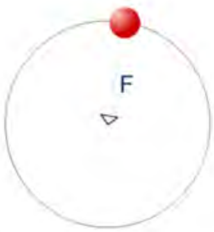
$\vec{v}_f = \underline{\underline{25\,m/s}}$

$B \rightarrow \vec{v}_f = \underline{\underline{18\,m/s}}$

Since an object in UCM maintains the same speed!

Centripetal Force

According to Newton's second law: where there's acceleration, there's a force. The force causing an object to move in a circle is called the centripetal force. This force is different than other forces we've looked at so far, because it really just a new way of thinking of other forces we've already studied, like gravity, tension or friction.



To find centripetal force, we use Newton's Second Law, subbing in the centripetal acceleration formulas for "a".

$|\vec{a}_c| = \frac{v^2}{r}$ $\quad \vec{F} = ma$ $\quad \boxed{\vec{F}_c = \frac{mv^2}{r}}$

$|\vec{a}_c| = \frac{4\pi^2 r}{T^2}$ $\quad \vec{F} = ma$ $\quad \boxed{\vec{F}_c = \frac{4\pi^2 r m}{T^2}}$

where: r = radius (m)
$\vec{v}$ = velocity (m/s)
m = mass (kg)
$\vec{F}_c$ = centripetal force (N)
T = period (s)

Note: while the centripetal acceleration formulas are shown on your data sheet, the centripetal force formulas ARE NOT shown.

I've never thought this was a big deal, because you should really know at this point in Physics 20 that in order to get a force, you must multiply acceleration by mass.

$$\left|\vec{a}_c\right| = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Friction

$F_c = \frac{mv^2}{r}$

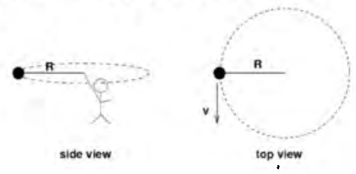
$F_c = F_f$

This centripetal force can be made equal to any force that makes an object move in a circle. Let's look at a few examples:

ex) A 3.0 kg brick is tied to a rope and swung in a horizontal circle. The rope has a length of 0.85 m and can withstand a maximum force of tension of 100 N before breaking. Determine the maximum speed the brick can move at before breaking the rope.

$\vec{F}_c = \vec{F}_T$

$\frac{mv^2}{r} = \vec{F}_T$



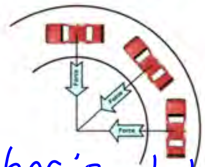
$\frac{(3.0kg)v^2}{0.85m} = 100N$

$v = \underline{\underline{5.3 m/s}}$

ex) Determine the maximum velocity a car can turn a corner of radius 50.0 m in dry weather (μ = 0.90) and in wet weather (μ = 0.10).

$\vec{F}_c = \vec{F}_f$

$\frac{mv^2}{r} = \mu \vec{F}_N$ recall: $\vec{F}_N = \vec{F}_g$ on a horizontal!



$\frac{\cancel{m}v^2}{r} = \mu \cancel{m}\vec{g}$

$\frac{v^2}{50m} = (0.90)(9.81 m/s^2)$

$v = \underline{\underline{21 m/s}} \leftarrow$ Dry

$v = \underline{\underline{7.0 m/s}} \leftarrow$ wet

Tomorrow: Vertical Circular Motion!