Physics 20 Unit B - Dynamics

Field Theory and Universal Gravitation

POS Checklist:

- ☐ identify the gravitational force as one of the fundamental forces in nature.
- ☐ describe, qualitatively and quantitatively, Newton's law of universal gravitation.
- ☐ explain, qualitatively, the principles pertinent to the Cavendish experiment used to determine the universal gravitational constant, G.
- ☐ define the term "field" as a concept that replaces "action at a distance" and apply the concept to describe gravitational effects.
- ☐ relate, qualitatively and quantitatively, using Newton's law of universal gravitation, the gravitational constant to the local value of the acceleration due to gravity.

The Tooth Fairy, theories and Pedagogical Lies.

(and how that relates to my kids and gravity)

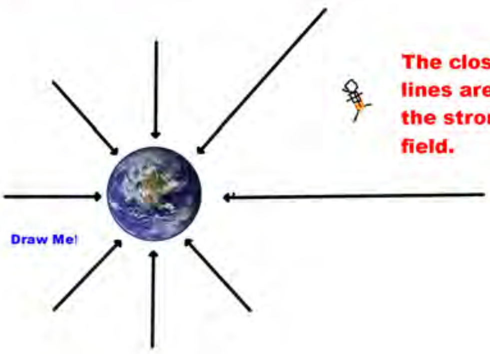
One way of explaining gravity:

Gravitational Field Theory

Invisible gravitational fields surround all objects at all times. Every mass in the universe creates its own gravitational field.

This theory is neither "true" or perfectly accurate, but it makes pretty good sense and is easy to understand.

Fields are represented by field lines, which are vectors. The vectors extend forever and always point towards the centre of objects.



The closer the field lines are together, the stronger the field.

We know what direction field lines will move in (towards the centre of objects), now here's how we can work out the magnitude:

$$\vec{g} = \frac{\vec{F}_g}{m}$$

where:

$\vec{g}$ = strength of gravity field (N/kg)
F_g = force of gravity on object (N)
m = mass of test object (kg)

*Note: this is just Newton's second law revisited.

If we release a test object in a gravity field, it will accelerate in the direction of the field with a force proportional to the mass of the object

ex) If a shark of mass 175 kg experiences a force of 1480 N towards Earth, what is the gravitational field at the shark?

$$\vec{g} = \frac{\vec{F_g}}{m} = \frac{1480\,N}{175\,kg} = 8.29\,\frac{N}{kg}$$

There is a second formula for finding the magnitude of gravitiational fields.

Let's see if you can guess a few properties of the gravitational field:

- if I have a box in space that creates a gravitational field of 100 N/kg. If I double the mass of the box, what will the gravitational field be?

Ans: N/kg

- if am 100 m away from that box, and the gravitational field is 100 N/kg, what will the field be when I am 200 m away?

Ans: N/kg

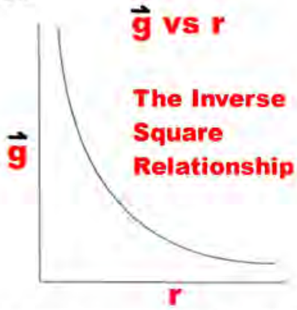
Experimentally, it has been found that the strength of the gravitational field is directly proportional to the mass of the object creating the field.

$$\vec{g} \propto m$$

This means that if we double the mass of an object producing a gravitational field, we double the field itself.

It was also found through experimental means (page 203) that the value was also inversely proportional to the square of the distance between the test object and the gravity producing object.

$$\vec{g} \propto \frac{1}{r^2}$$



http://www.makingthemodernworld.org.uk/learning_modules/maths/06.TU.02/?section=6

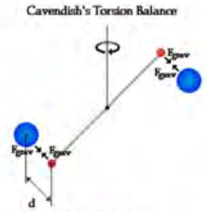
We can combine these findings to make our second formula:

$$\vec{g} = \frac{Gm}{r^2}$$

*

$\vec{g}$ = gravitational field strength (N/kg)
G = universal gravitational constant
m = mass of gravity producing object
r = distance between the center of the test object and the center of the gravity producing object

The constant G was proposed by Newton and finally calculated some years later through the results of **Henry Cavendish's Cavendish experiment** (which was used to determine the mass of the Earth).



See pg. 205-206

http://www.learnalberta.ca/content/sep20/movieResourceLauncher.htm?movie=smil/fundamentalforces_cavendish.smil

The value of G was calculated from Cavendish's data to be:

$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

Universal Gravitational Constant

This value is quoted on your formula sheet and is the same everywhere in the universe!

Also remember that this is an exact value and is not subject to sig-digs.

Universal Gravitational constant (data sheet)

mass of Earth (data sheet)

ex) Calculate the gravitational field strength on the surface of the Earth.

*Note: you will need to use some values from your data sheet here!

$$\vec{g} = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(5.97\times10^{24}kg)}{(6.37\times10^{6}m)^2}$$

radius of Earth (data sheet)

$$\vec{g} = 9.81\frac{N}{kg}$$

ex) Calculate the gravitational field strength on the highest peak of Everest (8848 m above the earth).

$$\vec{g} = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(5.97\times10^{24}kg)}{(6.37\times10^{6}m + 8848m)^2}$$

$$\vec{g} = 9.786\ N/kg$$

ex) Calculate the gravitational field strength on the lowest point of the Marianas Trench (11034 m below the surface of the earth).

$$\vec{g} = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(5.97\times10^{24}kg)}{(6.37\times10^{6}m - 11034m)^2}$$

$$\vec{g} = 9.8475\ N/kg$$

Proportionality Problems

ex) Calculate gravitational field strength on the planet LDtopia, whose mass is 1/8 that of Earth and whose radius is three times larger.

Step 1: write out the eqn.

Step 2: replace the = with a $\propto$ and cancel out anything that doesn't change.

Step 3: sub in a number for the changing quantity.

$$\vec{g} = \frac{Gm}{r^2}$$

$$\vec{g} \propto \frac{(1)(\frac{1}{8})}{(1)^2}$$

$$\vec{g} \propto \frac{1}{8}$$

$$\vec{g} = \frac{1}{8} \times 9.81\ N/kg = 1.23\ N/kg$$

when we "cancel out", we place a 1 in place of the variable.

Newton's Law of Universal Gravitation

Newton not only worked out a formula for field strength: he also combined this formula with his second law to determine the Law of Universal Gravitation.

$$\vec{g} = \frac{Gm}{r^2} \quad \text{and} \quad \vec{F}_g = m\vec{g}$$

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

where:

$\vec{F}_g$ = force of gravity between objects 1 and 2

G = Universal Gravitational Constant

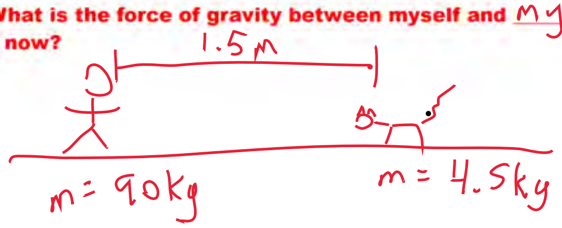
m_1 = mass of object 1

m_2 = mass of object 2

r = distance between centres of objects

mass of this is on data sheet

ex) What is the force of gravity between myself and my cat **right now?**



$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

$$\vec{F}_g = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(90kg)(4.5kg)}{(1.5m)^2}$$

$$\vec{F}_g = 1.2\times10^{-8}N$$

ex) What is the force of gravity between two neutrons placed 150 pm apart?

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

$$\vec{F}_g = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(1.67\times10^{-27}kg)(1.67\times10^{-27}kg)}{(150\times10^{-12}m)^2}$$

$$\vec{F}_g = 8.27\times10^{-45}N$$

ex) If the distance between two objects doubles, and the mass of the objects stay the same, what can be said of the force of gravity between them?

$$\vec{F}_g = \frac{Gm_1m_2}{r^2} \qquad \vec{F}_g \propto \frac{(1)(1)(1)}{(2)^2}$$

← If a variable stays the same, we put a 1 in the proportionality eqn.

← This is how we show "doubling" separation

$$\vec{F}_g \propto \frac{1}{4}$$

← Our answer means the new force is one quarter of the old force.

HW:

Questions: Page 215 #3 - 6

ex) If the distance between two objects halves, and the mass of one of the objects stay the same while the other triples, what can be said of the force of gravity between them?

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

$$\vec{F}_g \propto \frac{(1)(1)(3)}{(\frac{1}{2})^2}$$

$$\vec{F}_g \propto \frac{3}{\frac{1}{4}}$$

$$\vec{F}_g \propto 12$$

← This answer means that the new force will be 12 times greater.