Math 10C: Unit 3: Exponents and Radicals

The Exponent Laws

POS Checklist:

3.1 Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

3.2 Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, $n > 0$.

3.3 Apply the exponent laws:

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}$, $a \neq 0$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$

to expressions with rational and variable bases and integral and rational exponents, and explain the reasoning.

Review:

1. Identify each number as a perfect square or a perfect cube using prime factorization.

a) 216 **b) 169**

2. Evalutate.

a) 3^1 =
b) 3^0 =
c) x^1 =
d) x^0 =

Shortcut:
- anything to the power of 1 = _____,
- anything to the power of 0 = ___.

Exponents

- are a simple way of showing repeated multiplication

ex) 2^5 = (2)(2)(2)(2)(2) = 32

base **exponent**

But there is a lot more we can do with exponents than just simple repeated multiplication...

Negative Exponents

What does it mean to have a negative exponent? Complete the following table to see.

$10^3 = 10 \times 10 \times 10 = 1000$
$10^2 =$
$10^1 =$
$10^0 =$
$10^{-1} =$
$10^{-2} =$
$10^{-3} =$

Negative exponents give reciprocals.

$$x^{-1} = \frac{1}{x}$$

The Negative Exponent Law

ex) Evaluate.

a) 2^{-2}

b) 5^{-3}

c) 1^{-7}

ex) Write with a positive exponent.

a) $x^{-3}y^6$

b) m^4n^{-6}

Multiplying Exponents:

We can also multiply exponents with a shortcut. Try this example and see if you can find the shortcut.

$(3^2)(3^3) = (3 \times 3 \times 3)(3 \times 3) = 3^{\square} = 243$

$(y^2)(y^3) = (y \times y \times y)(y \times y) = y^{\square}$

$(z^3)(z^{-2}) = \dfrac{(z \times z \times z)}{(z \times z)} = z^{\square}$

Shortcut:

$$(a^m)(a^n) = a^{m+n}$$

When multiplying exponents of the same base, add the powers.

ex) Simplify.

a) $(5^2)(5^5) =$

b) $(a^2)(a^3) =$

c) $(b^{-10})(b^5) =$

d) $(x^3)(y^2) =$

Dividing Exponents:

We can also divide exponents with a shortcut. Try this example and see if you can find the shortcut.

$$\frac{4^3}{4^2} = \frac{(4)(4)(4)}{(4)(4)} = 4^{\square}$$

$$\frac{x^5}{x^3} = \frac{(x)(x)(x)(x)(x)}{(x)(x)(x)} = x^{\square}$$

Shortcut:

$$\frac{(a^m)}{(a^n)} = a^{m-n}$$

When dividing exponents of the same base, subtract the powers.

ex) Simplify.

a) $\dfrac{(7^2)}{(7^5)} =$

b) $\dfrac{(a^2)}{(a^3)} =$

c) $\dfrac{(b^{-10})}{(b^5)} =$

d) $\dfrac{(x^3)}{(y^2)} =$

Power of a Power Rule

See if you can find the shortcut:

$(4^3)^2 = (4^3)(4^3) = 4^{\square}$

$(x^5)^3 = (x^5)(x^5)(x^5) = x^{\square}$

Shortcut:

$$(a^m)^n = a^{mn}$$

When taking the power of a power, multiply the exponents.

ex) a) $(x^5)^4 =$

b) $(x^{-3})^4 =$

Bracket Rules

Try and work out these last two shortcuts:

$$(xy)^3 = (xy)(xy)(xy) = x^{\square}\, y^{\square}$$

$$\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^{\square}}{y^{\square}}$$

Shortcut:

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

The Bracket Rules

ex) a) $(xy)^4$ =

b) $(x^3z^5)^2$ =

c) $(x/y)^4$ =

d) $(x^2/y^5)^3$ =

e) $(x^3/z^2)^{-3}$ =

Homework: page 169 #2, 4, 5, 6 every other letter, 14.